

# Technical Paper for BOAT3D ---- Planing Boat Time-Domain Simulation with Waves

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## **Introduction and Summary**

This is a 4th, updated 2008 version of the technical paper originally written in 2003 and discussing simulation software BOAT3D. This paper pertains to Version 4.2, the 2003 paper pertained to Version 3.0, the 2004 paper referred to 3.1, and the 2007 paper referred to 4.0.

Payne (1), (2), (3), (4) and earlier workers (see history in reference 2) have developed an added-mass approach to planing boat behavior and other aerodynamic/hydrodynamic phenomena. Early history of this development goes back as far as 1924 with the work of Munk (5). In developing a mathematical theory of boat behavior, primary attention in the literature has been focused on smooth-water resistance and trim, etc. However, a planing boat is normally exposed to waves, and its utility depends on its performance in rough water. The BOAT3D simulation computer program developed by Payne (6) has facilities for running computations with waves. BOAT3D provides a mathematical model which can be implemented on a PC. It thus does not contain highly detailed physics computations (which might require a super-computer) but rather various engineering approximations tested by comparing its results to experiment.

In 2002-2003 major changes were made to BOAT3D. Once significant modifications had been made it was desirable to test the results of the program against experimental data. A preliminary set of trials was done, and the calculated boat behavior generally agreed with the experimental data, as discussed in the 2003 paper.

In 2004 the primary goal was to improve the features of the BOAT3D program, to make it more user-friendly. However, new data (11) were obtained for higher-speed planing, and some "fine tuning" adjustments were made to Version 3.1 to improve the agreement with both old and new data. The 2004 paper was the report on that work.

In 2007 the primary goal was to remove restrictions on the shape of the boat to be analysed. This resulted in major changes to the internal structure of the program (to obtain version 4.0) which are not very much visible in the user interface. Again fine tuning was done. The earlier comparisons with experimental data have been repeated to ensure that the BOAT3D program remains effective.

Version 4.1 (spring 2008) only involved user interface changes, and there was no update of this paper.

For version 4.2 a boat shape option of "double chines" was added. A number of internal changes were made to allow a more logical handling of cases involving the double chines. Also several errors were fixed, and it was found that in some unusual or seldom-used cases, illogical and erroneous results could occur, so additional changes in the 4.1 internals of the program were made to give Version 4.2. Only very slight changes occurred in the comparisons with tank tests recalculated for this paper, but major changes were made in the comparisons with the sea trials of Blount and Hankley (9).

## **Brief Introduction for Added-Mass Approach to Planing Boat Analysis, and Description of Modifications in going from versions 2.x to 3.x, to 4.0, to 4.2**

### **Added-Mass Basic Concepts:**

When a boat is traveling at planing speeds the mathematical model implemented in BOAT3D predicts that the main force acting on the boat is due to the effects of "added mass", although there is also a

buoyancy force computed, and certain semi-empirical adjustment factors may be applied. According to the added-mass approach, a region of water under the boat (the added-mass) is assumed to move as a solid body being pressed downward by the motion of the boat, whereas the rest of the water ("bulk of the water") is assumed unaffected by the boat's presence. Of course, this is an approximation; the actual situation is that the boat motion creates a continuous flow field under the boat. There are theoretical supports for the added-mass approach, but there is a large degree of empiricism in it. We attempt to create a simulation model which reasonably mimics experimental results.

For the added-mass analysis, consider a slice of water  $\Delta x$  thick extending across underneath the boat as the boat passes over it (Figure 1). As the boat moves along its bottom surface and the added mass in the slice of water is moving downward at a rate  $v_{\text{boat}}$  and the bulk of the water (excluding the added mass) is moving up or down at a rate  $v_{\text{water}}$  because of wave action (Figure 2). According to the "slender boat" assumption conditions are not changing very rapidly along the length of the boat, so we neglect any motion of the water in the  $x$  (fore and aft) direction.

(In BOAT3D,  $x$  is the fore-and-aft direction,  $y$  is beam-wise, and  $z$  is vertical.)

Figure 1 Moving Boat, Stationary  $x$  Coordinates

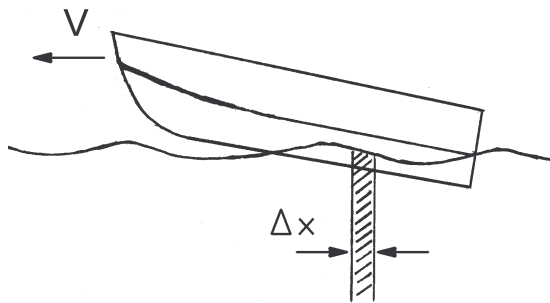
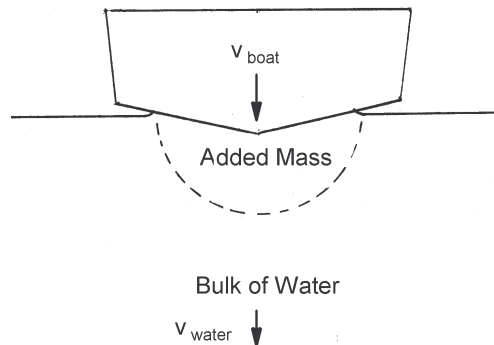


Figure 2 Schematic of Added Mass



According to the added-mass assumption, the force exerted on the boat by the water slice is equal to the rate of change of momentum of the added mass.

Force of  $\Delta x$  thick slice of water on boat = Rate of change of momentum of added mass

$$F = \Delta x \frac{d}{dt} (m' \Delta v) \quad (\text{Eqn 1})$$

where  $m'$  is the added mass per unit thickness of water slice, and  $\Delta v = v_{\text{boat}} - v_{\text{water}}$ .

Frequently formulas of the general form

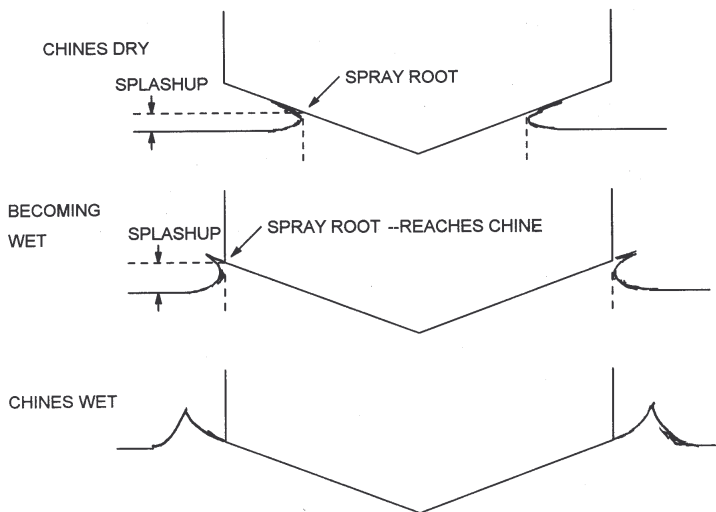
$$m' = C_m (\pi/2) \rho y^2 \quad (\text{Eqn 2})$$

are used to obtain the value of  $m'$ . In the newer versions of BOAT3D, the general types of formulas developed by Payne (2, 4) continue to be used, although in version 3.1 it was found that increasing the dependence of  $C_m$  on deadrise gave improved results, and in version 4.2 there are some additional changes.  $y$  is half the wetted width (i.e.  $y$  is the radius of dashed circle in Figure 2) and  $C_m$  is a factor depending on boat shape.  $\rho$  is the density of water. The wetted width is wider than the width of the part of the boat below the undisturbed water line because of "splashup" i.e. water is pushed upward and outward as the impact of the boat raises the water pressure.

For a more precise description of "splashup" one needs to examine the flow in detail as is done by Vorus in Reference 13. The "spray root" discussed by Vorus is called "stagnation point" in some BOAT3D internal documentation, although "spray root" is the more correct terminology.

Figure 3 shows the concepts of "spray root" and "splashup" as used in connection with BOAT3D.

Figure 3 "Spray Root" and "Splashup" as a boat cross-section descends into the water



The figure shows three stages as a boat cross-section descends into i.e. impacts the water. "Chines Dry" is the condition where the spray root has not yet reached the chine. (Some water in the spray jet may have already touched the chine but we still call it "dry".) In this stage the added mass is rapidly growing (as is the dimension  $y$  in Eqn 2) leading to an increasingly large upward dynamic force applied to the boat. When the spray root reaches the chine the added mass stops growing rapidly, and the dynamic force

suddenly drops to a low value. (See Figure 5 below for examples of dynamic force vs time.) After the "Chines Wet" state has been reached we are no longer dealing with a splashup or an increasing  $y$ , and the added mass is computed primarily based on how far the boat cross-section has moved downward since the "Chines Wet" state was reached.

### Change of Coordinate System:

To obtain the overall force on the boat it is necessary to sum over all slices of water which it is contacting, and to do this, it is desirable to convert to a coordinate system moving along with the boat. The new coordinate  $X$  is moving relative to  $x$  with boat forward velocity  $V$ .

With conversion of the coordinate system, the force equation becomes (by the chain rule)

$$F = \Delta x \left[ \frac{\partial}{\partial X} (m' \Delta v) \frac{\partial}{\partial t} X + \frac{\partial}{\partial t} (m' \Delta v) \right]$$

which is

$$F = \Delta x \left[ V \frac{\partial}{\partial X} (m' \Delta v) + \frac{\partial}{\partial t} (m' \Delta v) \right] \quad (\text{Eqn 3})$$

For a boat traveling on smooth water, the second term in the brackets is small or zero, but with waves, it is not negligible. In versions of the BOAT3D program previous to 3.0, this term was always neglected. Putting it into the 3.0+ versions of BOAT3D has increased greatly the computational effort needed to compute boat behavior. However, in the time gap between versions 2.3 and 3.0 of BOAT3D computer speeds have increased greatly, offsetting (but only partially) this difficulty. The comparisons with data discussed below indicate that the current BOAT3D approach is producing reasonable results.

In addition to the above change in BOAT3D added-mass approach for the newer versions, vertical and horizontal velocities of the waves are no longer neglected. At the surface of waves the water is moving up and down with the full velocity needed to produce the travel of the waves on the surface, but the vertical velocity diminishes approximately exponentially with depth. Because there was some indication that using the surface vertical velocity might give too large a wave effect on boat behavior, a heuristic approach is currently used in BOAT3D, and the wave velocity at a depth of some fraction of the  $y$  (half-beam) dimension is assumed. For the results presented below, a depth of 20% of  $y$  was assumed. Results are not very sensitive to this depth factor, but 20% gives reasonable results. This is used in all newer versions.

### Dropping the "Shed Added-Mass" force

In the added-mass formulations used in BOAT3D, the added mass always increases as the boat cross-section is inserted deeper into the water and correspondingly always decreases as the boat section is withdrawn from the water. We find that in modeling conventional planing boats running at planing speeds the withdrawal motion seldom ever happens and is slight if it happens. However, with a semi-planing design (much rocker) running at moderate speeds we noticed that the versions 3.0 through 4.1 formulation predicted a highly excessive up-force at the stern where withdrawal was taking place. Expanding equation (1),

$$F = \Delta x \left( m' \frac{d\Delta v}{dt} + \Delta v \frac{dm'}{dt} \right) \quad (\text{Eqn 4})$$

we assume the following behavior in version 4.2:

For increasing  $m'$  (and  $\Delta v$  is positive in this case, i.e. insertion of the boat cross-section deeper into the water) equations 1 and 4 apply as written. However, for decreasing  $m'$  (withdrawal with both  $\Delta v$  and  $dm'/dt$  negative) the second term in equation 4 is assumed NOT to contribute to the force  $F$ , i.e. it is dropped from the equation.

Our explanation for this is that the second term represents change in momentum of "shed" added mass no longer associated with the boat, so it is dissipated to the body of water and not to the boat. Clearly this explanation is not based on rigorous physics, but it removes the erroneous upward force on the boat.

In fact, in a simulation of one of the "boat drop tests" of Reference 14 (not performed with BOAT3D but with BOAT3D equations in DMSolver) if the second term in equation 4 is **not dropped** for withdrawing from the water, the energy is transferred back into the dropped boat section, which bounces back up to the height from which it was dropped. This is clearly not realistic, and when the second term in equation 4 is **dropped** during withdrawal the boat section only bounces a slight amount.

This new assumption for version 4.2 is somewhat analogous to the old assumption in versions 2.x that added mass force  $F$  is 0 wherever  $m'$  decreases along the length of the boat (the Jones (7) assumption as discussed by Payne (2, p. 43)). The old assumption was not used in versions 3.0 through 4.1 because it does not consider the change in coordinate system and gives illogical discontinuities in force distribution along the boat (sections of 0 force with every little wave), and these discontinuities occurred in practically every case with waves.

With the new assumption the second term in equation 4 dwindles gradually to zero as a positive  $\Delta v$  decreases and then is kept at zero arbitrarily as  $\Delta v$  goes negative, so there is no abrupt discontinuity.

A negative  $\Delta v$  does not occur very often in typical planing boat analyses, so this major change in equations does not affect published results computed with versions 3.0 through 4.1. As mentioned above, it has a major effect for certain unpublished tests for boats with large rocker or boats moving very slowly.

### Semi-Planing Froude Numbers:

In the past, with versions 3.0 to 4.1, we have found that BOAT3D results agree closely with results of Fridsma (Reference 8) for his highly prismatic tank test models (e.g. Figure 6) down to the semi-planing speed having length-based Froude number  $F_n=0.6$  (i.e. Speed/length ratio=2, see below). However, some of our unpublished test results done in the development of V.4.2 have shown that BOAT3D was underestimating some results and overestimating others compared with experiments with more normal boat designs (i.e. with lower, more normal prismatic coefficients in the semi-planing range, say  $F_n=0.6$  to  $F_n=1.2$ ). Specifically, because BOAT3D does not (because of the slender boat assumption) take into account wake wave fore-and-aft propagation, it tended to underestimate trim rise and resistance in the semi-planing range. Also, it tended to overestimate the "bounciness" of the boat, or to be more properly technical, it underestimated the damping of the heave motion at semi-planing Froude numbers.

For V.4.2 we have used the same "artificial low speed damping" built into the program by Peter Payne, but the values used for the damping coefficient are computed differently to give better agreement with

experiment. The damping force is proportional to the buoyancy force times the vertical velocity relative to the water. The proportionality factor used to vary as  $(\text{length})^{-0.5}$  and go to zero at high Froude numbers. It is now varying as  $(\text{length})^{-0.1}$  and is not arbitrarily cut off at high Froude number (at which damping force decreases as a normal thing because dynamic force is proportional to velocity squared and buoyancy force dwindles).

Fixing the underestimation of trim and resistance at low Froude numbers must wait until a future version of BOAT3D since doing this while retaining all the other features of the program is difficult.

### **"Dynamic Suction":**

Payne (reference 2, starting at page 49) developed the idea that there is a reduction in lift to below that predicted by the added-mass analysis, and he referred to this effect as "Dynamic Suction". He applied this reduction near the stern, which tended to add to the trim angle of the boat. A Dynamic Suction factor was used to reduce the buoyancy force near the transom. Since the buoyancy force is overshadowed by the added-mass force at high speeds, Dynamic Suction had little effect at high speeds.

For Version 3.0 equations in the program were changed so that somewhat less Dynamic Suction force was developed. This gave improved results, but with further testing it was found that predicted trim angles tended to be too low at higher speeds when the Dynamic Suction was adjusted to give good agreement at lower speeds. To help with this problem, in versions 3.1+ the Dynamic Suction adjustment has been applied to both buoyancy and added-mass forces, not just to buoyancy.

Although the Dynamic Suction adjustment magnitude is decided by empirical means, its existence is theoretically reasonable at all speeds. For a planing boat moving fast enough to have a dry transom there is free water surface (pressure exactly atmospheric) right where the flow leaves the bottom of the transom. Higher pressure under the boat is reduced near the transom because there is acceleration of water (in violation of the "slender boat" assumption) from a higher pressure region toward the lower (atmospheric) pressure right at the transom.

In addition to the Dynamic Suction adjustment to added-mass and buoyancy forces, BOAT3D continues to employ the small adjustment to added-mass forces for aspect ratio discussed by Payne (4). Both of these adjustments are approaches to compensate for real-world violation of the "slender boat" assumption.

### **Extension to a Wider Range of Shapes:**

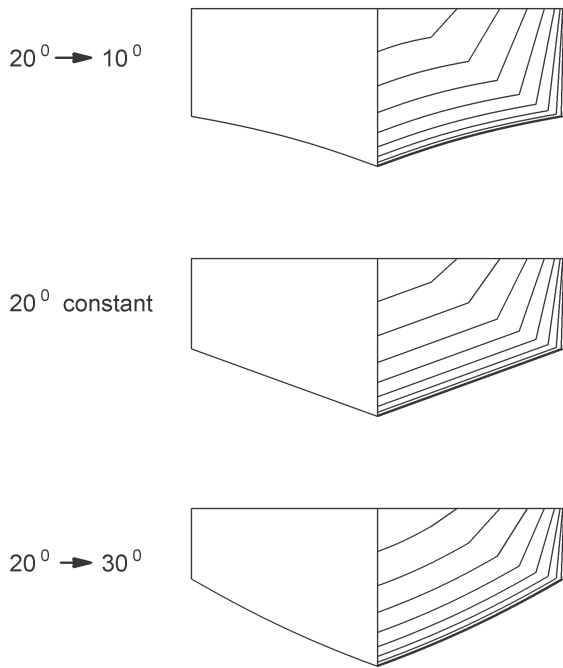
The goal in Version 4.0 was to allow for concave or convex bottom cross-sectional shapes which frequently occur in real boats, i.e. we do not want to continue to require a "straight-vee" i.e. wedge bottom shape. In previous versions of the program the slice of boat and water to which the added mass analysis was applied was always assumed to be vertical (as drawn in Figure 1). Since the boat was constantly changing pitch angle this meant that the shapes of the boat slices which figured in the calculation of added mass (e.g. the  $C_m$  factor, etc) were constantly changing, and  $C_m$  and other items were constantly having to be recomputed for each slice at each time step. This was only a little burdensome since the straight-vee formulas were simple, but with anything more complex (for a non-straight-vee case) the complexity was overwhelming. Therefore, for versions 4.0+, the values of added mass, and many other things depend only on the height reached by the water on a particular slice of boat, and a table of values is computed before actual time integration starts (this makes integration computing faster). The slices necessarily are fixed to

the boat coordinates and not always vertical as assumed in Figure 1 and equations 1 and 3. Comparisons of results showed that this does not significantly change the results from BOAT3D. It is simply another use of the low pitch angle assumptions which have always been in BOAT3D to some extent, and the effects of certain other geometrical approximations (about the wave profiles being relatively flat) have been reduced in versions 4.0+.

### The Effect of Non-straight-vee (Curved) Bottom Cross-sections

Unfortunately we do not have access to any detailed research studies which can be used to validate experimentally a BOAT3D taking into account a non-straight-vee bottom. Our approach to this problem is based on theoretical results of Vorus (13). This author presents a theoretical, approximate method for computing the impact force on a non-straight-vee bottom, and presents results for 3 comparable cases in which a slender boat impacts the surface at constant velocity. We prepared three cases corresponding very closely (but not exactly) to Vorus's.

Figure 4 Cases with Specified Deadrise Angle at the Keel and at the Chine



Starting with Equation 1 with constant velocity and using the chain rule

$$F/\Delta x = v (dm'/dt) = v (dm'/dz)(dz/dt) = v^2(dm'/dz) \quad (\text{Eqn 5})$$

Vorus presents results for a coefficient of normal force  $C_f$  where

$$C_f = \frac{F/\Delta x}{\rho v^2 y_{chine}} \quad (\text{using BOAT3D notation, } y_{chine} = 1/2 \text{ beam at chine}) \quad (\text{Eqn 6})$$

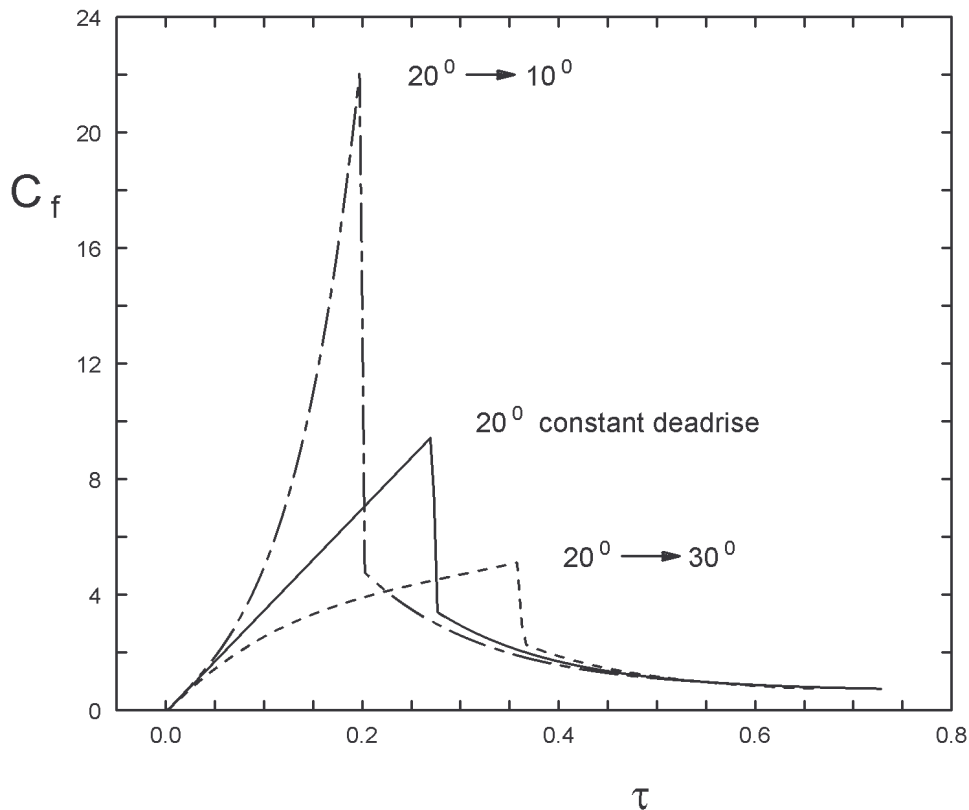


This is presented as a function of dimensionless time

$$\tau = t / (v y_{chine}) \quad (\text{Eqn 7})$$

$C_f$  vs  $\tau$  can be easily computed from the information that BOAT3D 4.x assembles for each boat slice (cross-section), as in Figure 5.

Figure 5 BOAT3D 4.2 Added-mass Force-per-unit-length Coefficient VS Dimensionless Time (for a slender boat impacting the surface at constant speed starting at  $\tau = 0$ )



This figure compares closely with Vorus's Figure 14. After the keel of the boat strikes the water initially at  $\tau = 0$  (and deadrise of 20 degrees), the force grows linearly with time when there is a constant deadrise, faster than linearly when the deadrise decreases to 10 degrees at the chine, and slower than linearly when the deadrise increases to 30 degrees at the chine. After the chine becomes wet the force suddenly drops off to a much lower value. The chine wetting happens soonest when the deadrise decreases (boat bottom is concave) and latest when the deadrise increases (convex bottom).

Rather than use Vorus's complex mathematical model, the above curves were generated by an evolutionary modification of the previous BOAT3D methodology, and still resemble Vorus's curves closely. There are several steps involving two equations in this method.

- (1) The nominal  $y$  (=half-beam) is the distance from the center line to the point on the boat's bottom intersecting the undisturbed water line.
- (2) The splashup factor  $\psi$  is computed based on the deadrise angle of a straight line from the keel to the point on the boat's bottom mentioned in step (1) i.e. at the undisturbed water line with half-beam nominal

y. We use a somewhat different equation from the traditional one (see below).

(3) The "stagnation point" is found from the splashup factor --- it is the point on the actual hull contour at the vertical splashup rise mentioned in step (2).

(4) The  $C_m$  factor for added mass is computed based on the deadrise angle of a straight line from the keel to the stagnation point. The same equation as in version 3.1 is used. (But if chines are wet we use a different approach than used in previous versions.)

(5) If the boat bottom is concave then the line mentioned in step (4) encloses some water between it and the bottom surface, and we use this enclosed area to compute an extra added mass of water which is added in the final step. Likewise, if the boat bottom is convex then we subtract excluded added mass of water in the final step.

The values of  $C_f$  and  $\tau$  at the instant before chine wetting (location of the peak in Figure 5) can be compared with Vorus's Figure 14.

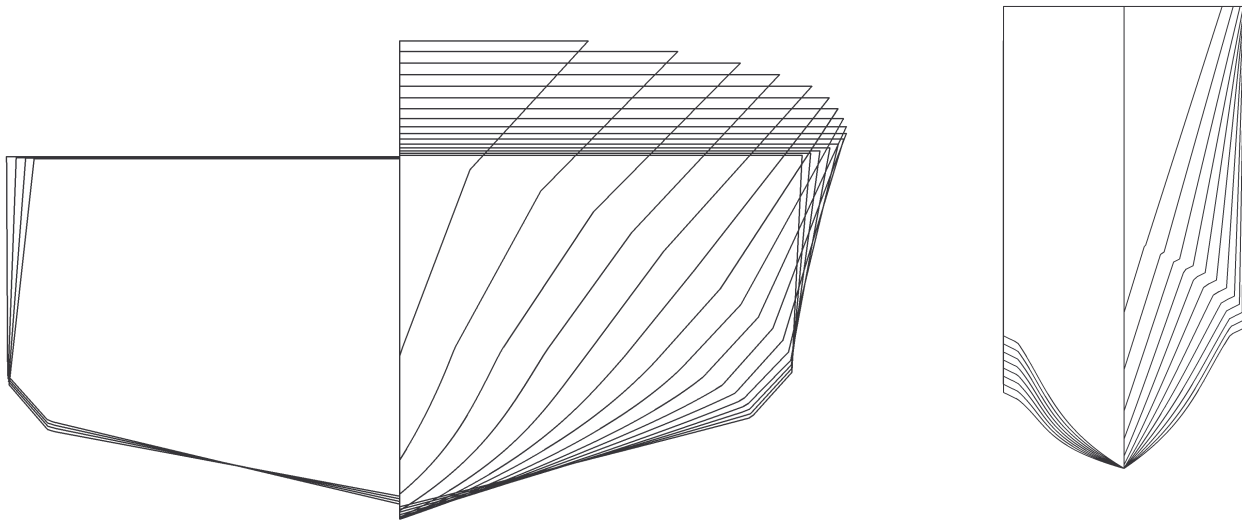
Case	Vorus $C_f$	BOAT3D $C_f$	Vorus $\tau$	BOAT3D 4.0 $\tau$
20-->10	19.0	22.0	0.209	0.197
20-->20	8.5	9.4	0.269	0.269
20-->30	5.1	5.1	0.344	0.358

After chine wetting a new equation is used for V4.2 which makes the  $C_f$  drop exponentially toward a constant value as the chines submerge deeper. This resembles the results of Vorus's analysis, whereas in previous versions a  $C_f$  which was constant (except for a step which was an artifact of splashup accounting) was used.

### The "Double-Chine" Shape Options:

These have been introduced for V4.2. There is an inner panel of the boat bottom starting at the keel and extending to the inner chine, and the panel can have the cubic polynomial shape of V.4.0 and V4.1 as described above. Then there is a straight segment extending to the outer chine, which is considered the "real" chine hydrodynamically, i.e. according to usual BOAT3D rules flow is always assumed to separate at the "real" chine (it may or may not separate at the inner chine).

The diagram on the left below shows the BOAT3D representation of Navy Patrol Boat "Craft I" in Blount and Hankley's paper, Reference 9. The diagram on the right shows a catamaran hull conceptual design. In the Craft I case (a real boat) dual chines are being used to round off hard chines, whereas in the conceptual catamaran (not an actual boat) they are intended to provide a sort of spray rail/lifting device.



It should be mentioned that the analytical work of Vorus that supports our assumptions about the cubic polynomial curved shape introduced in V4.0 involves relatively mild deviations from the straight-vee (i.e. wedge) shape whereas the user can by using the cubic polynomial and double chines specification methods input boats which deviate drastically from the straight-vee shape. We caution the user that we have not been able to compare BOAT3D results with experimental results in such cases, except for Craft I presented below. Reference 14 shows a case (a drop test of a boat section with horizontal "lifting strakes"--their Figure 6) for which the Ref. 14 authors do not have a good mathematical model--the spike of force which they predict as the strake hits the water is much higher than they measured, and in their actual experiment the spike gets smoothed out over a longer period of time. Likewise, if the user defines the double-chine panel to be a spray strip with nearly zero deadrise in his BOAT3D case, BOAT3D will very probably overestimate the height of the lift force spike as the double chine becomes wet because in the real-boat-case the slender-boat assumptions get violated, smoothing out the spike. Since there is effectively smoothing of force over time when integration produces the motion prediction of the boat, BOAT3D's acceleration prediction will be much more in error than its prediction of the path of the boat.

### Equation Details:

The "traditional" equations which were used in BOAT3D versions 2.3 and 3.0 are (see reference (3))

$$\text{Splashup: } \psi = \pi/2 - \beta(1 - 2/\pi) \quad \text{where } \beta \text{ is the deadrise angle (Pierson's hypothesis).}$$

$$\text{Added mass: } C_m = (1 - 0.5 \beta/\pi)^2$$

NOTE: In references 3 and 4 the Pierson's hypothesis splashup equation is stated instead as

$$1 + \psi = \pi/2 - \beta(1 - 2/\pi)$$

so the definition of  $\psi$  was changed accordingly between these instances. In BOAT3D source code  $\psi$  is the

vertical distance from the keel up to the spray-root (i.e. wetted) point divided by the vertical distance from the keel up to the undisturbed water level. By the BOAT3D definition  $\psi$  is 1 if there is no splashup above the undisturbed water level, by the reference 3 & 4 definition,  $\psi$  is 0 if there is no splashup.

In versions 3.1+ the following change was made because slightly better agreement with data was obtained:

$$\text{Added mass: } C_m = (1 - 0.8 \beta/\pi)^2$$

and this change was kept in versions 4.0 and 4.1 but the following additional change was made:

$$\text{Splashup: } \psi = \pi/2 - \beta + (2\beta/\pi)^{1.3}$$

and then in 4.2 this has become

$$\text{Splashup: } \psi = (\pi/2)(1 - (2\beta/\pi)^{0.5}) + (2\beta/\pi)^{0.45}$$

which still has the desired properties that at  $\beta = \pi/2$ ,  $\psi = 1$  (no splashup) and at  $\beta = 0$ ,  $\psi = \pi/2$  ( $\pi/2$  is the standard limiting result in the literature).

## **Use of Tank Test Models vs. Full-Sized Boats**

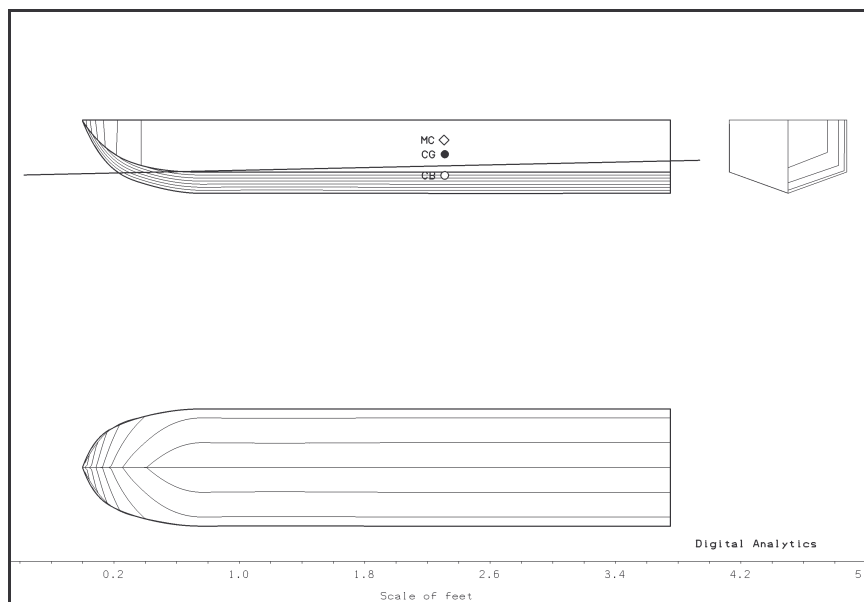
Most of the comparisons presented below in this report are with tank tests of small models. This is simply because easily obtainable published results of full-sized boat tests are few and lack the complete set of details required for a BOAT3D run. In contrast, the tank test studies by Fridsma (8) and Katayama et al (11, 12) provided all details required for BOAT3D. These studies involved artificially-generated uniform-height (and well-characterized) waves.

However, it is comforting to see close agreement between simulation results and results of full-sized boat tests for the Navy patrol boats I and II, even though various assumptions had to be made about details as discussed below.

## Comparison of Simulation Results with Data of Fridsma for Uniform Waves

Fridsma (8) conducted tank tests of model boats in uniform, single-frequency waves at the Davidson Laboratory and measured the boat's periodic response. The boat shapes used were prismatic with elliptical bows, and 10, 20, or 30 degrees of deadrise (see Figure 6). He varied boat length-to-beam ratio (but this was primarily 5). He varied boat weight (but this was primarily 16 lb), and trim

Figure 6 The 20-Degree Deadrise Model



angle (primarily either 4 or 6 degrees). Test runs were conducted at speed-to-length ratios of 2, 4, and 6 knots per  $\text{ft}^{1/2}$ . Several wave heights, and many wave length-to-boat-length ratios were used. Altogether results for 102 test runs in uniform waves were published. Results published included resistance (i.e. drag), 2 x amplitude of vertical motion oscillations due to waves, 2 x amplitude of trim oscillations due to waves, and peak vertical acceleration due to waves. Fridsma also published results of many runs on flat calm water for the same set of boat models.

18 of Fridsma's regular wave runs were selected for this study and BOAT3D runs were made corresponding to them. Only the two smallest wave-length-to-boat-length ratios (1 and 2) were used since longer waves are less of a test for the simulation. (Fridsma showed that for long waves the boat tends to ride the wave contours more precisely, so there is less relative motion between boat and water.) BOAT3D runs were made for all 3 deadrisers at 4 degrees trim and 4 and 6 knots per  $\text{ft}^{1/2}$  speed-length ratios. For 20 degrees deadrise additional BOAT3D runs were also made to show the effect of a higher (6 degree) trim, a higher wave height, and slower speed (2 knots per  $\text{ft}^{1/2}$ ). The 4 and 6 degree trim values are "nominal"; the actual trim oscillates with the waves and the mean of the oscillation is only approximately 4 or 6 degrees. It appears that Fridsma established (by interpolation) the CG position which would give the desired trim in flat calm water, and then used this CG position with waves. Fridsma did not publish his actual mean experimental trims, so we do not know how close these are to the stated values.

BOAT3D-calculated resistance, 2 x amplitude of vertical motion oscillations due to waves, 2 x amplitude of trim oscillations due to waves, and peak vertical acceleration due to waves were compared with the published experimental data (or values converted from Fridsma's experimental table using the

appropriate conversion factors). Table 1 shows results from the 18 runs. The case id code (e.g. A12) uses the letter code of Fridsma designating a particular deadrise angle/trim/speed and two numbers for wave height/wave length (A12 is 20 degree deadrise/4 degree trim/4 speed-length ratio/1 inch wave height/2 boat lengths wave length). Although Fridsma refers to 2 x amplitude, since the oscillations are not precisely sine waves, we actually add two different amplitude values, the upper amplitude of the peaks and the lower amplitude of the valleys.

Table 1  
Details -- Comparison of Uniform Wave Simulation Results With Experiment

Case Code	Wave Height (ft)	Wave Length (ft)	Deadr (deg)	X pos of CG (ft)	Spd/ (Lng) <sup>1/2</sup> kts/ft <sup>1/2</sup>	Resistance (lb)		Heave 2x Ampl. (ft)		Trim 2x Ampl. (deg)		CG Accel Peak (g's)	
						Sim.	Expt.	Sim.	Expt.	Sim.	Expt.	Sim.	Expt.
A11	0.0833	3.75	20	2.23	4	2.84	2.91	0.011	0.015	0.89	0.88	0.24	0.25
A21	0.1667	3.75	20	2.23	4	3.20	3.33	0.020	0.028	1.45	1.44	0.75	0.75
A12	0.0833	7.50	20	2.23	4	2.91	2.94	0.060	0.07	3.03	3.16	0.21	0.25
A22	0.1667	7.50	20	2.23	4	3.65	3.71	0.110	0.122	5.50	5.40	0.78	0.85
B11	0.0833	3.75	20	2.34	6	3.43	3.60	0.014	0.013	0.60	0.48	0.38	0.50
B12	0.0833	7.50	20	2.34	6	3.63	3.70	0.057	0.053	2.54	2.16	0.68	0.80
C11	0.0833	3.75	20	2.32	2	1.89	2.02	0.007	0.006	0.77	1.04	0.06	0.05
C12	0.0833	7.50	20	2.32	2	1.89	1.90	0.067	0.073	3.95	3.96	0.11	0.10
E11	0.0833	3.75	20	2.46	4	2.77	2.78	0.016	0.017	1.07	1.04	0.37	0.45
E12	0.0833	7.50	20	2.46	4	2.81	2.86	0.077	0.067	3.90	3.36	0.39	0.45
I11	0.0833	3.75	10	2.21	4	2.53	2.51	0.016	0.0175	1.08	1.12	0.56	0.60
I12	0.0833	7.50	10	2.21	4	2.57	2.61	0.063	0.063	3.04	3.12	0.30	0.25
J11	0.0833	3.75	10	2.56	6	2.88	2.91	0.018	0.019	0.72	0.64	1.22	1.20
J12	0.0833	7.50	10	2.56	6	2.96	2.99	0.056	0.047	3.07	2.48	2.18	1.35
K11	0.0833	3.75	30	2.3	4	3.00	3.09	0.0087	0.0108	0.71	0.72	0.12	0.20
K12	0.0833	7.50	30	2.3	4	3.13	3.14	0.064	0.060	3.28	2.80	0.22	0.30
M11	0.0833	3.75	30	2.27	6	4.03	4.38	0.009	0.011	0.47	0.48	0.19	0.30
M12	0.0833	7.50	30	2.27	6	4.21	4.45	0.049	0.049	2.06	1.84	0.36	0.45

Figure 7 shows the results for 2 x amplitude of heave (vertical) oscillations of the center of gravity. Simulation values are plotted against experimental values to create a "scatterplot". A perfect match lies on the 45 degree line. For this measurement points tend to group according to wave conditions. Each point in a group of 8 points (note K11 and M11 are superimposed) in the lower left corner represents a case with small wave height (1 inch) and the short wave length (1 boat length=3.75 ft). This group contains A11, B11, C11, E11, I11, J11, K11, and M11. One point slightly to the right/up from these represents A21, the only case with 2 inch wave height and 1 boat length wave length. In the second group toward the middle of the graph each point represents a case with 1 inch high/2 boat lengths long waves (A12, B12, C12, E12, I12, J12, K12, and M12). Finally, the single point toward the upper right represents the case A22 with 2 inch height/2 boat lengths long waves.

Figure 7 Double-Amplitude of Heave (Vertical) Oscillations of Center of Gravity, Simulated Values vs Experimental Values

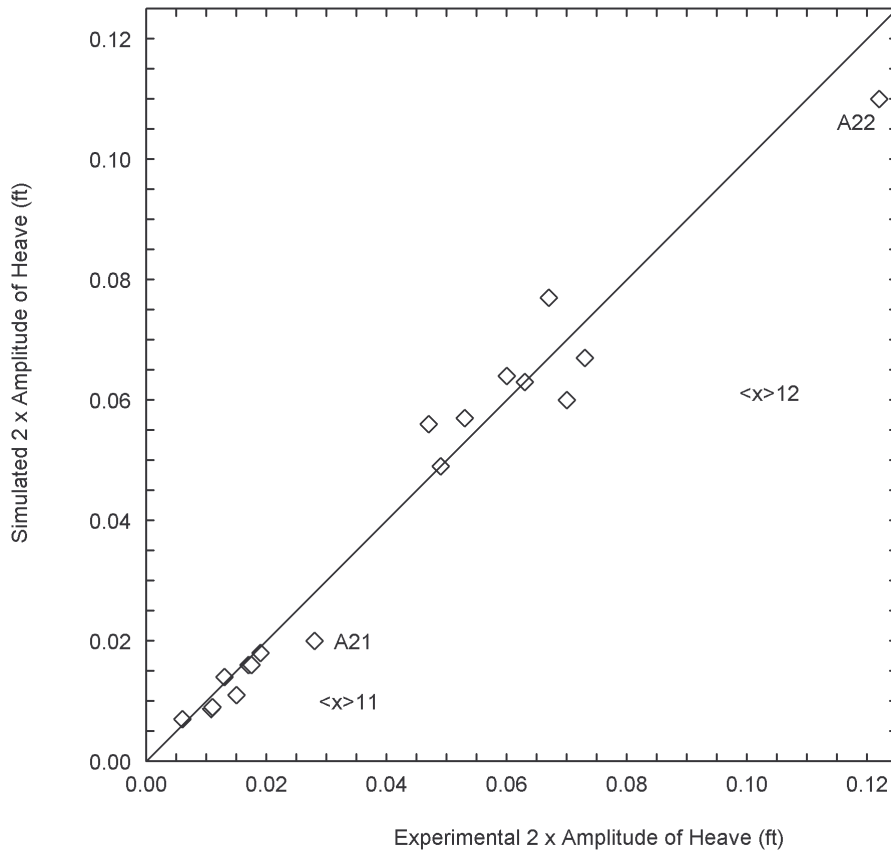


Figure 8 shows the results for 2 x amplitude of the trim oscillations. The points are grouped in the same way as those in Figure 7, although the middle group is more spread out.

Figure 8 Double-Amplitude of Trim Oscillations, Simulated Values vs Experimental Values

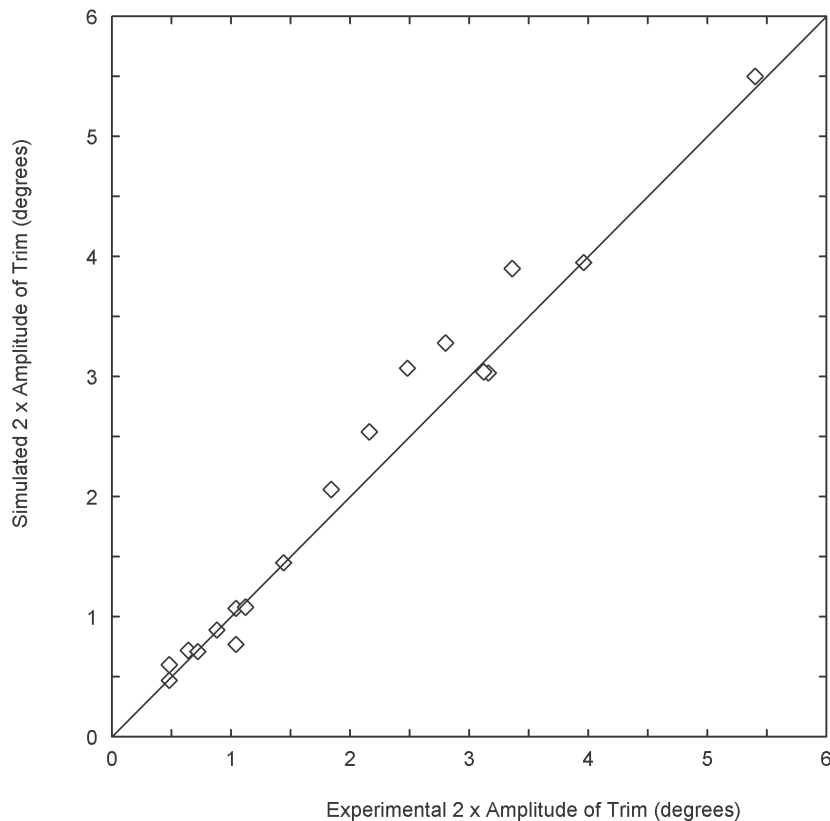


Figure 9 shows the results for absolute value of peak acceleration at the center of gravity. The two highest-acceleration points are for the two high-speed 10 degrees deadrise cases. 3rd, 4th, and 5th highest are for the two cases with 2 inch high waves (all others are for 1 inch waves) and the one case with high speed, 20 degrees deadrise, and long wavelength.

For boats encountering severe conditions, the upward, positive acceleration peak as the falling boat impacts the water is frequently more than 1 g. The negative acceleration peak (before the falling boat impacts the water) is limited to no more than -1 g and is always less in absolute value than the positive peak. However, in very mild conditions the simulation shows a reversed situation where in some cases the negative peak is somewhat bigger in absolute value than the positive peak. However, because Fridsma uses the positive peak we always report the positive peak. (In the last version of this report the negative peaks were used when they had bigger absolute values.)

The highest-acceleration case shows considerably larger simulated acceleration peaks than were measured by Fridsma. These higher peaks are very sharp spikes in acceleration produced by impact of the boat on the water. They are difficult for the Solver to simulate; very tiny integration time steps must be taken, and in the worst cases, the Solver still shows a scatter in peak heights. If they were present in the experiment, they may have been filtered out by the measurement procedure. Figure 10 shows a comparison between simulated acceleration traces for the highest acceleration case J12 and a lower acceleration case A11. (These were run with BOAT3D version 3.0, but version 3.1 and 4.2 give similar results.) Intermediate-magnitude acceleration cases also show spikes in acceleration; it was noticed that small adjustments in BOAT3D can make sizeable changes in spike heights even though heave and trim amplitudes only change slightly.



Figure 11 shows the results for resistance in lb force. In the simulation resistance is computed as an instantaneous value, and it fluctuates with wave action as does acceleration, trim, etc. There is an oscillating fore-and-aft acceleration due to the waves. (This was also true in the Davidson Laboratory tests because of the special towing carriage used.) In the results presented in Figure 11 the resistance force was computed by smoothing the oscillating force over 2 seconds to give a much smaller oscillation, and then visually finding the midpoint of the smaller oscillation on the graph.

Figure 9 Peak Accelerations at the CG, Simulated Values vs Experimental Values

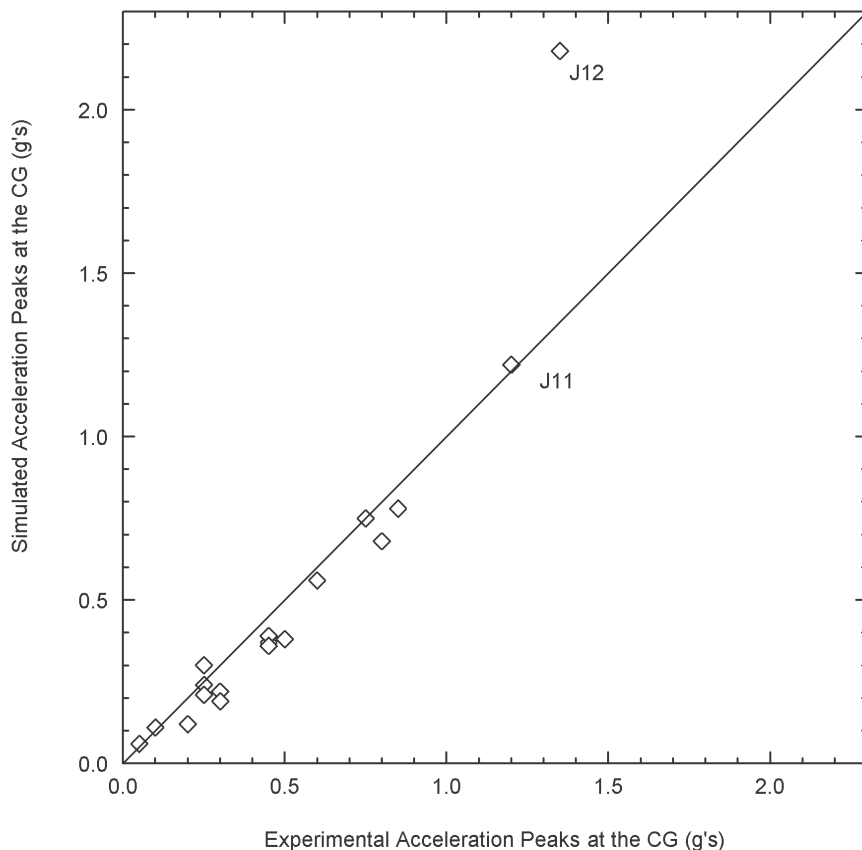
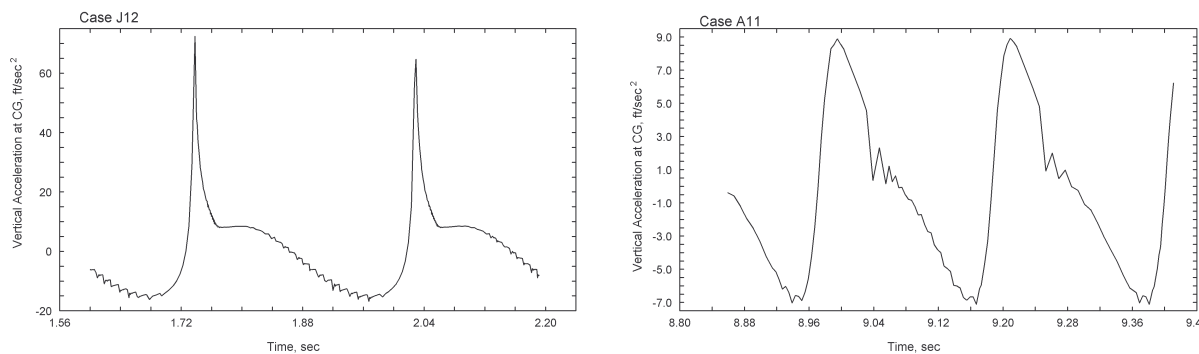
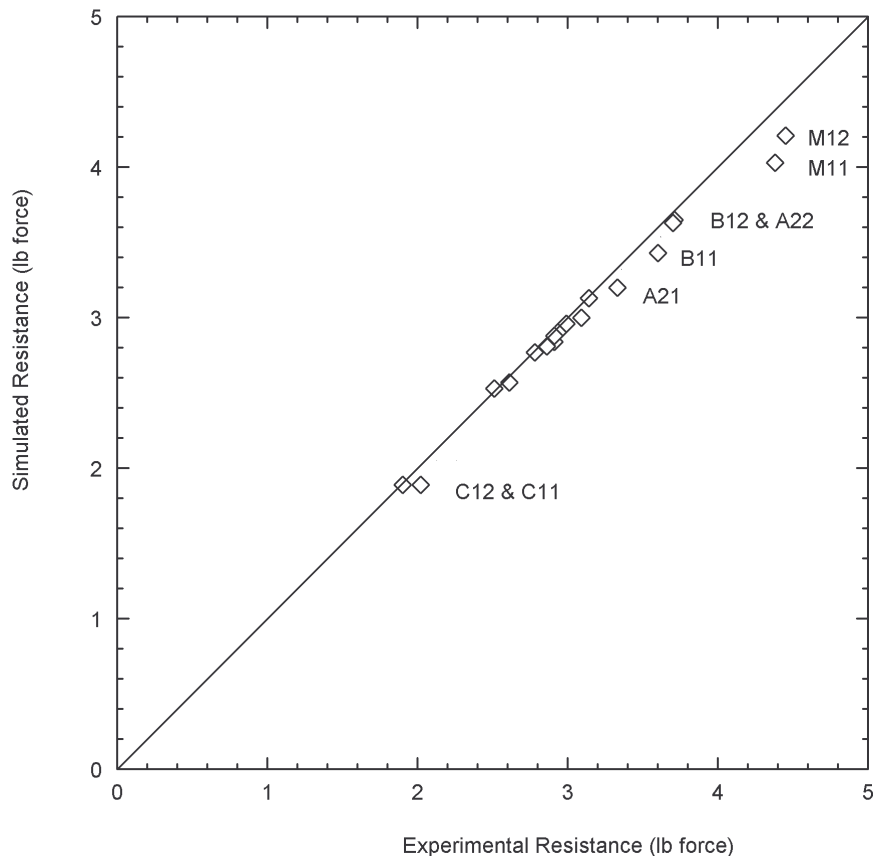


Figure 10 Comparison of Large and Small Acceleration Peaks



As can be seen when the simulated resistance differs from the experimental it tends to be slightly less. The increase in resistance with wave action compared with flat calm water tends to be bigger for the test tank model compared with the mathematical simulation.

Figure 11 Resistance in lb force, Simulated Values vs Experimental Values



### Comparison of Simulation Results with Data of Fridsma for Calm Water

Prior to producing the above-described results with waves, Fridsma conducted many preliminary tank tests with the same models in flat calm water. With waves the models were driven at their centers of gravity by a constant horizontal force (using the special carriage), and so had three degrees of freedom. For calm water the models were driven at constant speed and so had two degrees of freedom. The 2-degree-of-freedom option of the BOAT3D program was used to simulate some of the calm-water runs. (With BOAT3D, instead of running at constant velocity a gradual acceleration over 200 sec was used, so one simulation run covered the whole range of speeds desired.)

Figures 12, 13, and 14 show comparison BOAT3D runs (lines) with Fridsma's calm water trim data (points) from his figures 6, 11, and 16. For deadrise angles of 10° and 20° Fridsma's CG location was 60%, 65%, or 70% of the boat length aft of the bow, giving different trims. For 30° deadrise a 74% case was also included. The BOAT3D results agree fairly well with Fridsma's data except at speed-to-length ratios less than about 1.8. This deviation at low speeds is not unexpected. BOAT3D is designed to model planing, and is not usable for displacement boats.

Figure 12 10° Deadrise Calm Water Trim: Tank Test Data (Points) vs BOAT3D Simulation (Lines)

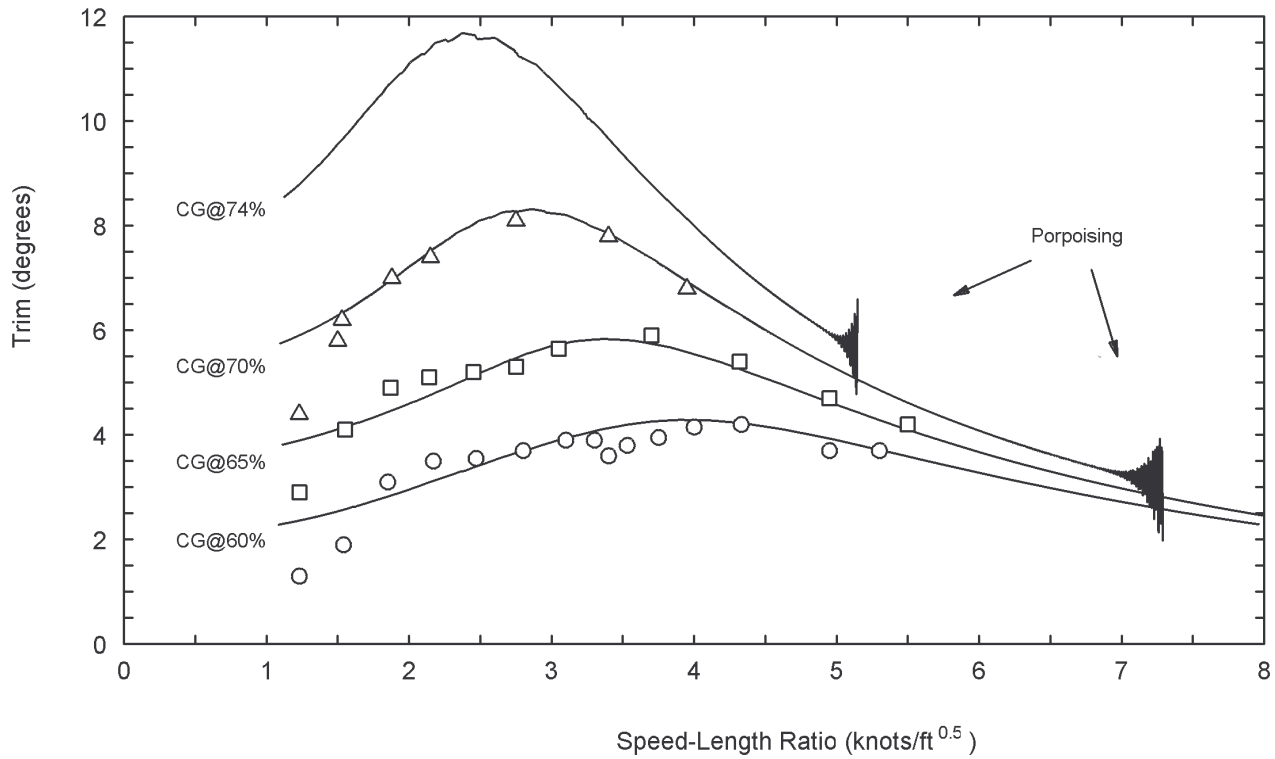
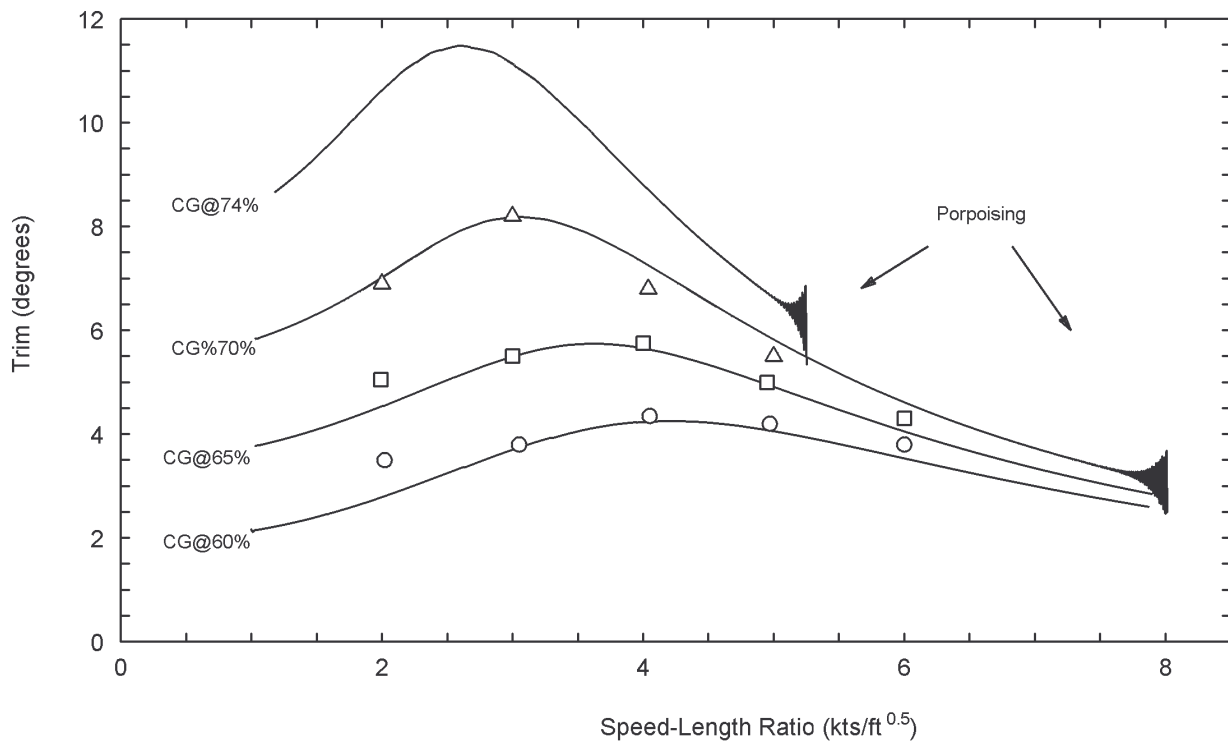


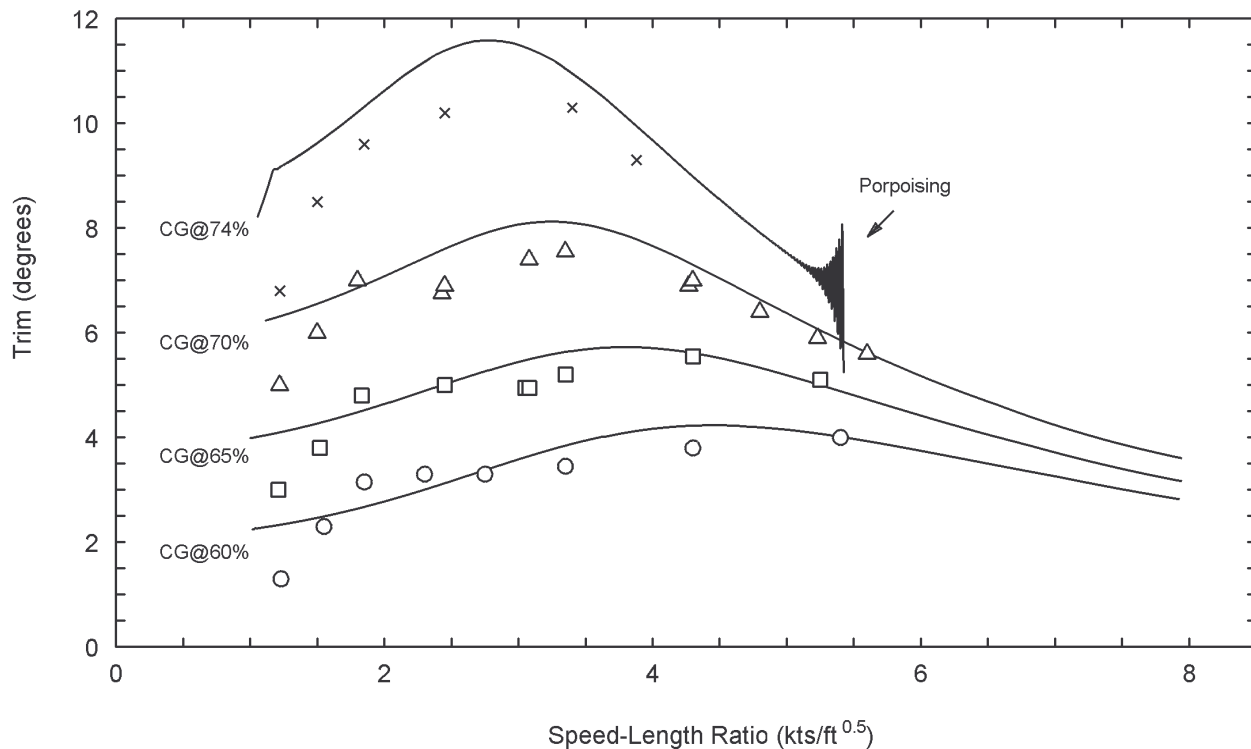
Figure 13 20° Deadrise Calm Water Trim: Tank Test Data (Points) vs BOAT3D Simulation (Lines)



In each of Fridsma's figures the highest trim case was noted to show porpoising instability at the higher speed-to-length ratios. BOAT3D runs for Fridsma's highest trim 10° and 20° deadrise cases were also found to exhibit porpoising instability, but at a higher speed than in the actual experiments. Additional

BOAT3D runs were done at higher trim angles to demonstrate instability. On figures 10 and 11 BOAT3D runs at 74% aft CG location are shown. These runs show porpoising instability at higher speeds.

Figure 14 30° Deadrise Calm Water Trim: Tank Test Data (Points) vs BOAT3D Simulation (Lines)



These tests on calm water show that BOAT3D Version 4.2 predicts somewhat more stability against porpoising than is actually present in the tank test. However, it is an improvement on version 3.1, which showed more stability yet. In contrast, the old Version 2.3 of the program showed much less stability than might ever be expected.

### Comparison of Simulation Results with Data of Katayama et al. for Uniform Waves

Katayama, Hinami, and Ikeda (11) performed tank tests on a model planing boat in calm water and uniform-amplitude waves. Compared with Fridsma, fewer combinations of parameters were used, with only one boat configuration (CG 54.4% aft of the bow reference point). However, much higher speed/length ratios were used. Fridsma's speed/length ratios of 2, 4, and 6 knots per ft<sup>1/2</sup> correspond to length-based Froude numbers of 0.59, 1.19, and 1.78, whereas Katayama et al. published a large number of results at Froude number 3.63 (a speed/length ratio of about 12.2 knots per ft<sup>1/2</sup>) and some Froude numbers are as high as 4.5. Other differences were that Fridsma used a special towing device giving 3 degrees of freedom, whereas Katayama et al. used a conventional 2-degrees-of-freedom arrangement. Katayama et al. did not publish resistance or acceleration.

Katayama et al's boat model has a constant deadrise bottom and resembles a model of a Jet-Ski. The method used to represent this boat's shape in BOAT3D is the Payne method with specified deadrise (see Figure 15).

Katayama et al's Figure 10 contains a set of their results for amplitudes of heave and trim response to waves as a function of wavelength. At a Froude number of 3.63 there were 3 wave heights (and many wavelengths) investigated. At a Froude number of 1.21 one wave height was investigated. The results are presented as "non-dimensional amplitudes". In figures 16 through 19 below the "Experimental" points were extracted from Katayama et al's Figure 10, and we call the values "Relative" rather than "Non-dimensional" amplitude.

For heave the "Relative" (i.e. "Non-dimensional") amplitude is the total distance the boat CG moves vertically (maximum minus minimum) divided by the wave height. For trim the Relative amplitude is the total trim change (maximum minus minimum) in radians divided by the (wave steepness times  $2\pi$ ), which is the same as total trim change in degrees divided by (steepness times 360). For a boat operating in very long wavelength waves, the boat accelerations are very small, the boat follows the wave contours exactly, and both relative heave and relative trim amplitudes approach 1. For a boat operating with the wavelength very much shorter than the boat length, the boat plows through the waves with little net effect, boat accelerations are small, and both relative heave and relative trim amplitudes approach zero.

Figure 15 Boat Model of Katayama et al (11) as Represented in BOAT3D

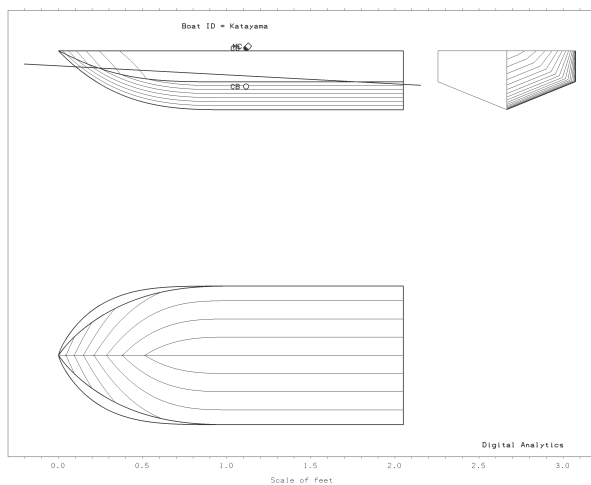


Figure 16 shows relative amplitudes for length-based Froude number of 1.21 (speed/length ratio of about 4.1). Thus these tests are at speeds comparable to those of Fridsma's tests. The wave height is 0.68 times the boat's nominal static draft of 0.059 m.

As expected, the boat motions tend to approach 0 at small wavelengths and approach relative amplitudes of 1 for large wavelengths. There are maximum relative amplitudes at intermediate wavelength values. The agreement of BOAT3D-simulated heave amplitudes with experiment is good except near the peak of maximum amplitudes. BOAT3D-simulated trim amplitudes tend to be considerably higher than experiment, and the experimental trim amplitudes for the two lowest-wavelength cases are not reported.

Figure 16 Relative Amplitudes of Motion with Froude No.=1.21, Wave Height 0.68 x Draft

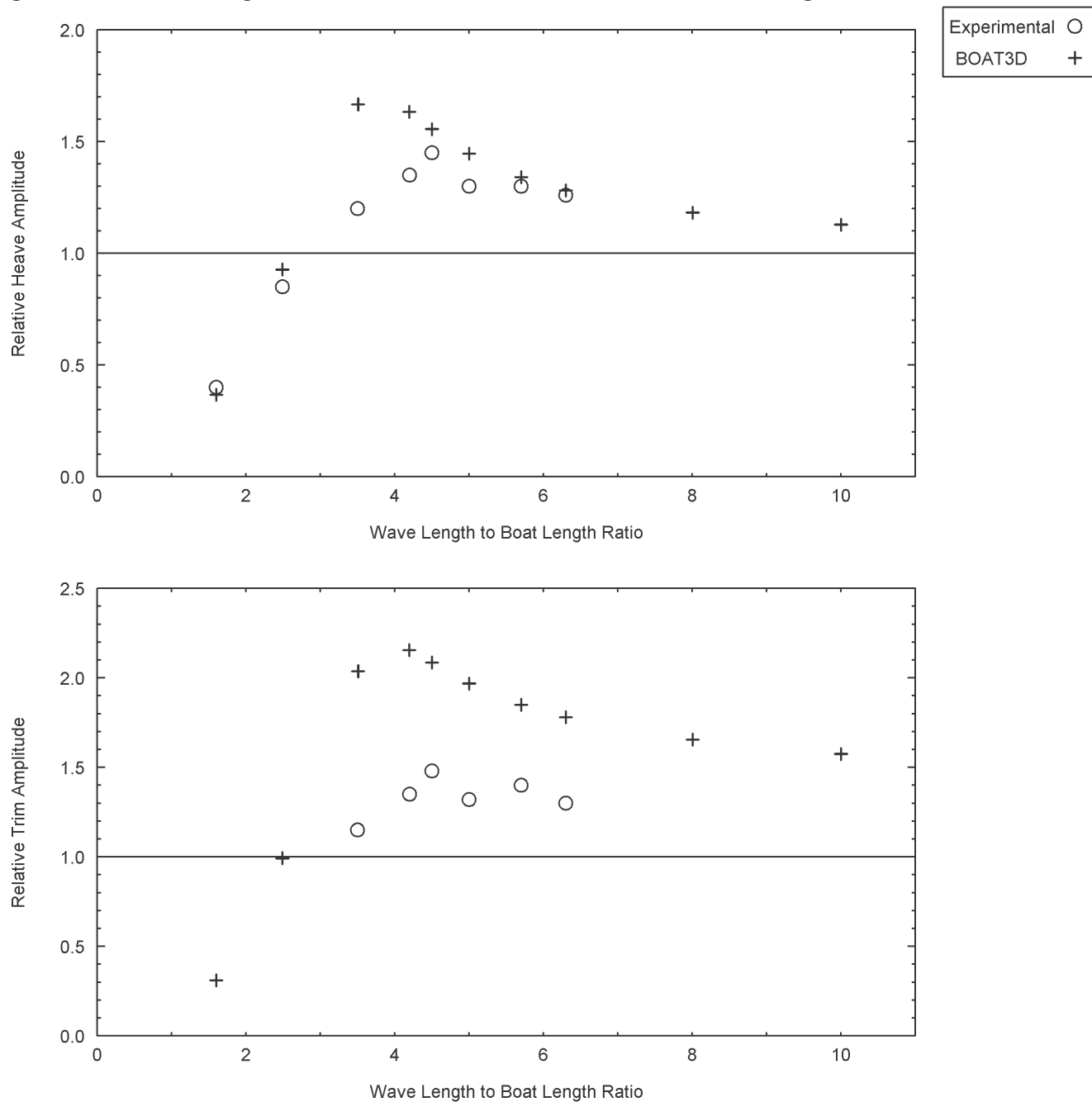
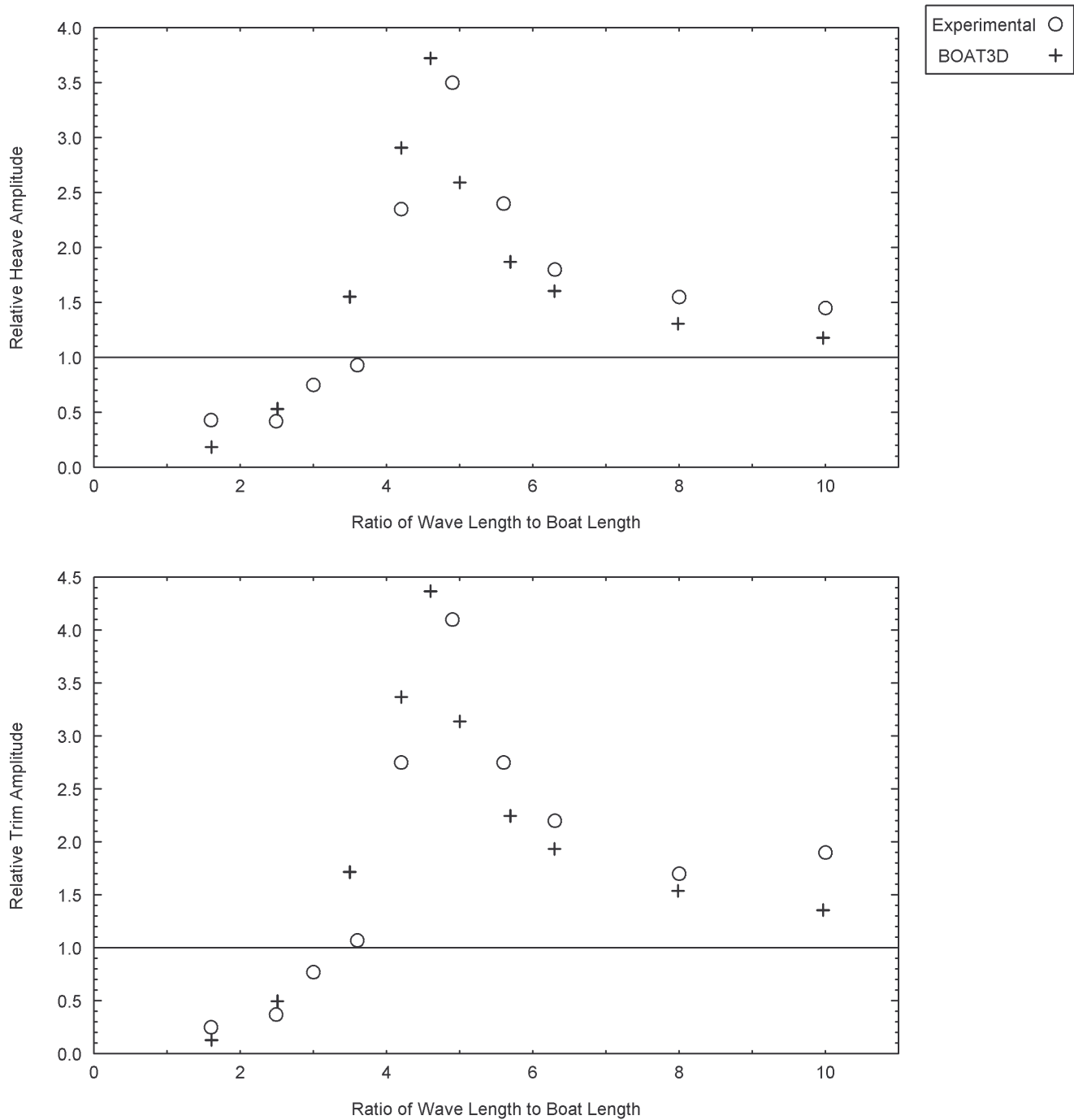


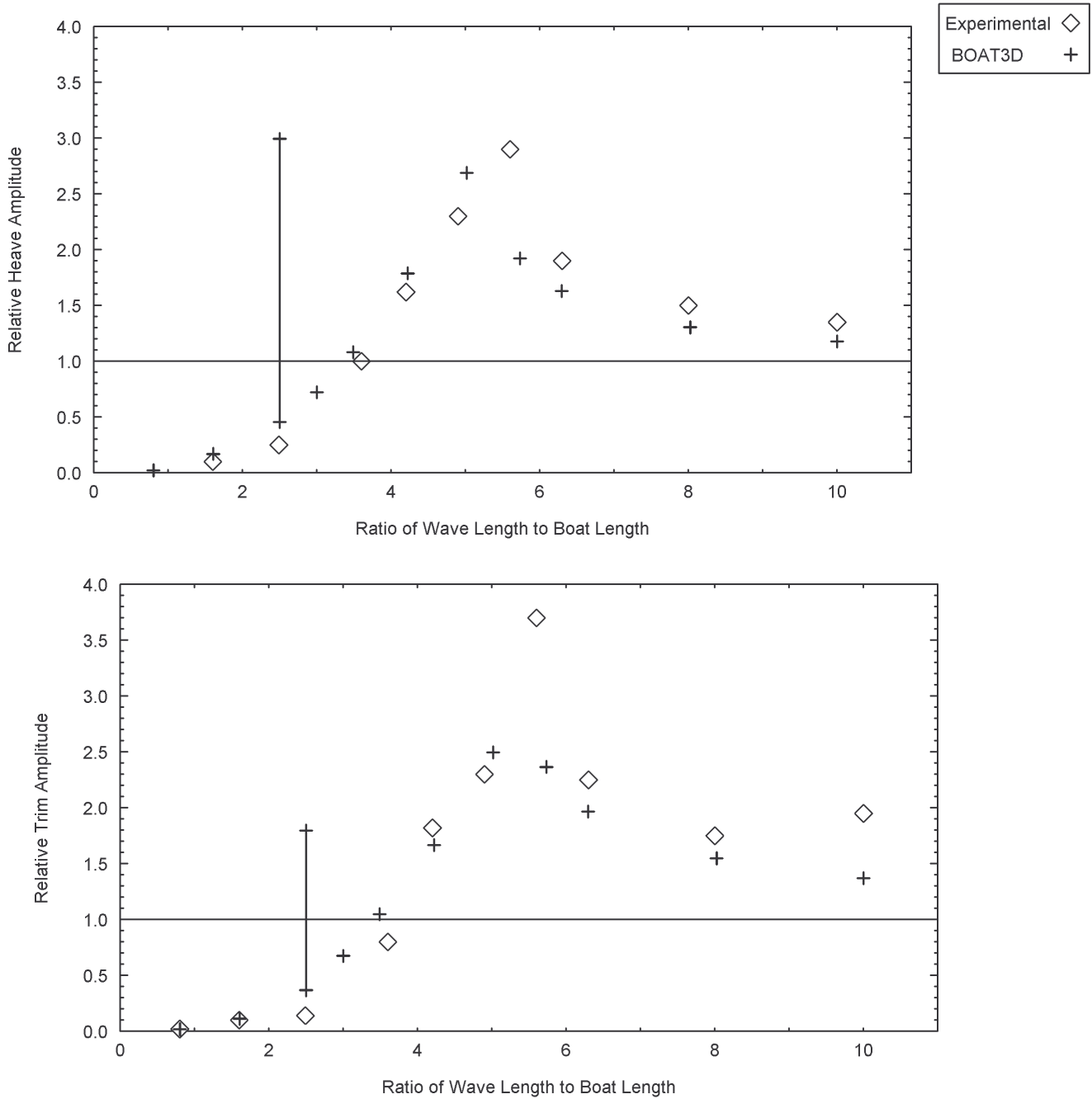
Figure 17 shows relative amplitudes for length-based Froude number of 3.63 (speed/length ratio of about 12.2). Thus these tests are at least twice the equivalent speed of any of Fridsma's tests. The wave height is 0.17 times the boat's draft. As expected, the boat motions tend to approach 0 at small wavelengths and approach relative amplitudes of 1 for large wavelengths. In this case at intermediate wavelength values the boat is leaping clear of the water from wave to wave. The experimental and BOAT3D-simulated amplitudes follow the same trend, but the experimental values tend to be larger at long wave lengths.

Figure 17 Relative Amplitudes of Motion with Froude No.=3.63, Wave Height 0.17 x Draft



The case depicted in Figure 18 is like that in Figure 17 except that wave height is doubled. Relative (but not absolute) amplitudes are reduced. Simulated amplitudes tend to be larger than experimental below the peak, and smaller at or above the peak.

Figure 18 Relative Amplitudes of Motion with Froude No.=3.63, Wave Height 0.34 x Draft

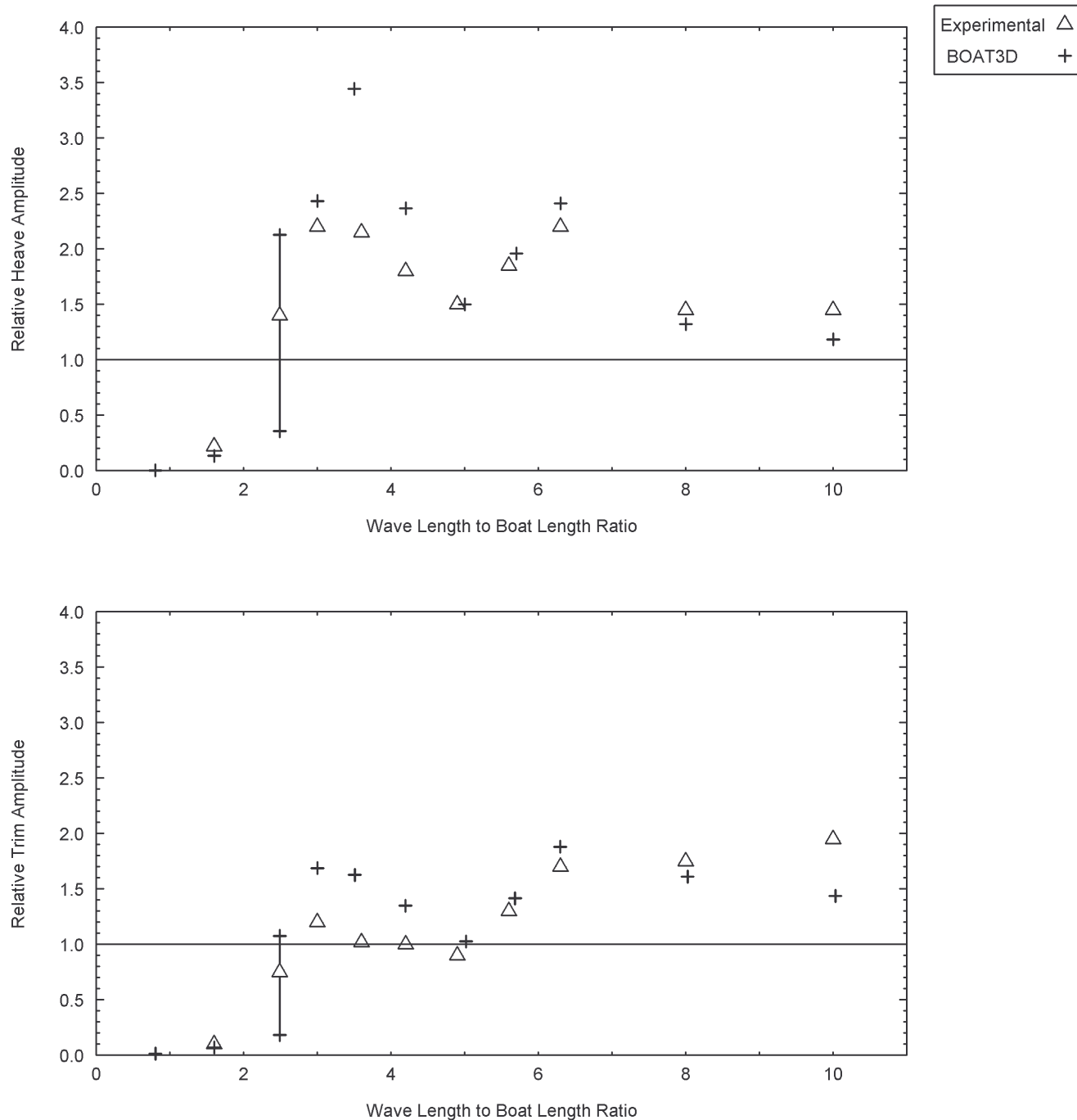


A strange effect occurs at a Wave/BoatLength ratio of about 2.5. The motion starts out having low amplitude and is in a mode where the boat strikes every wave almost equally. Then the motion switches to a persistent mode with the boat almost leaping over every other wave, with very much larger amplitudes and accelerations. We plot two points connected with a bar for the two modes. In at least one case whether we got the small or the large amplitude mode initially depended on initial conditions. In other cases we noted the opposite kind of transition, with larger-amplitude alternation of waves switching rather quickly to uniform motion striking every wave evenly (the final, stable mode which was reported).



In the case depicted in Figure 19 there is again a doubling of wave height compared with the previous case. With this case there are two peaks, at approximately 3 and 6 wavelength-to-boat-length ratios. Over the range of wavelength-to-boat-length ratios there are regions of good agreement, points where the simulated amplitudes exceed the experimental, and regions where the experimental amplitudes exceed the simulation results.

Figure 19 Relative Amplitudes of Motion with Froude No.=3.63, Wave Height 0.68 x Draft

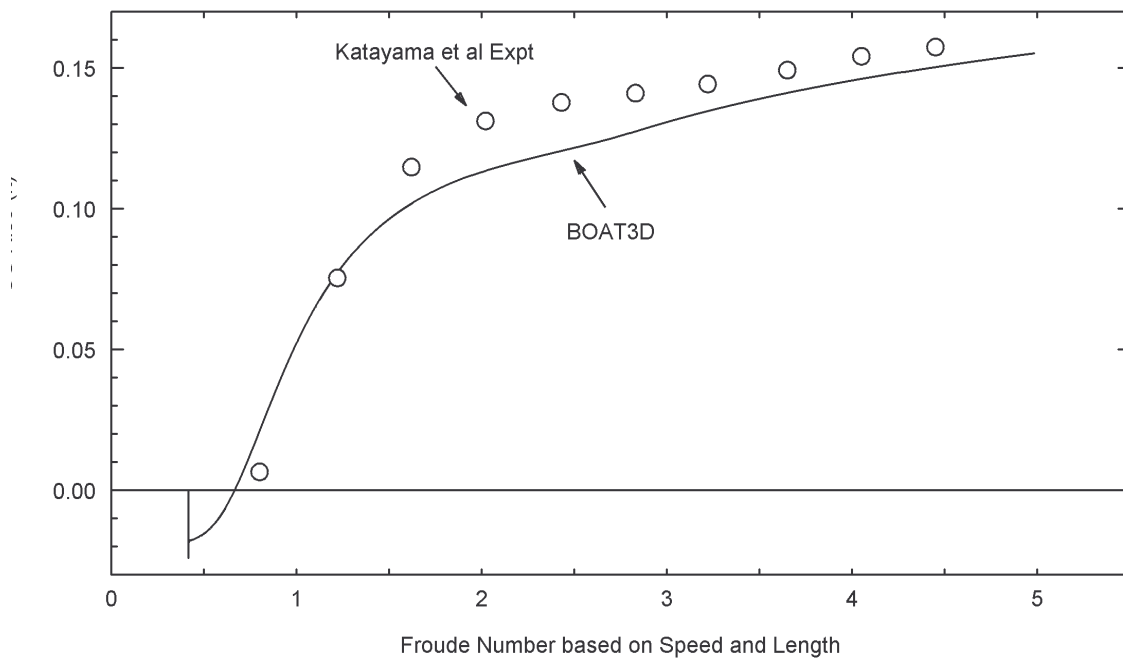
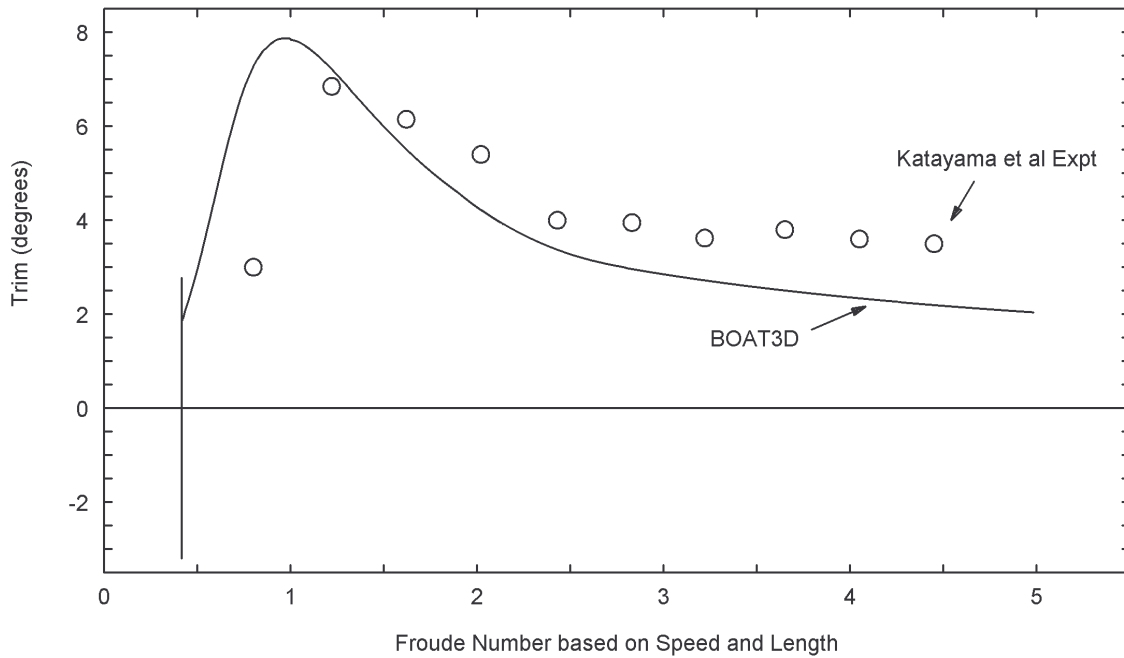


The same types of multiple mode occurrences were noted as with the Figure 18 runs.

## Comparison of Simulation Results with Data of Katayama et al. for Calm Water

A BOAT3D simulation run was computed corresponding to Katayama et al's Figure 4. This comparison is shown in Figure 20 below.

Figure 20 Comparison of BOAT3D Simulation with Results of Katayama et al. for Running Attitude in Calm Water (their Figure 4)



These results show relatively good agreement between experiment and simulation for heave amplitudes. However, for high Froude numbers (e.g. 4.5) the experimental trim required to produce lift is almost twice that computed in the simulation. The experimental model boat is running with a somewhat smaller wetted

bottom area than the simulated boat, and it is operating at almost double the angle of attack.

## Comments on Model Tests

In comparing BOAT3D results to experimental model tank test particular attention has been paid to (a) amplitudes of trim and heave in response to uniform waves, and (b) trim on flat calm water as a function of speed. This was done because both experimental studies used for comparison provided these data. Also, acceleration due to wave impact and hence ride comfort is to some extent related to these amplitudes, and resistance is to some extent related to average trim angle. Of course, when there are sharp peaks in acceleration the height of the peaks is very sensitive to small phase differences between trim and heave responses, and peak heights are hard to predict.

In comparisons of BOAT3D amplitudes of trim and heave with experimental data relatively good agreement is obtained at length-based Froude numbers below 2, i.e. operating conditions for typical cruising and utility planing boats. For very high speed sport boats and racers (Froude number  $>2$ ) mixed results are obtained. In some cases where the boat is airborne much of the time there can be big differences in the results. We discovered multiple modes of motion under these conditions, so that if the simulated boat and the tank-test boat happened to be in different modes, disagreement between mathematical model and physical system can be large even when both are operating correctly.

In comparisons of BOAT3D with experiment for trim as a function of speed in calm water, again good agreement is obtained at lower speeds, except that the threshold for spontaneous porpoising motion is somewhat higher with BOAT3D than in experiment. High trim angles and relatively fast speeds promote porpoising. Fridsma's 10 degree deadrise boat with its CG set 70% back from the bow porpoised at a Froude number of about 1.3, whereas the BOAT3D run with CG at 70% porpoised at about 2.1. Fridsma's 20 degree deadrise boat with its CG set 70% back porpoised at a Froude number of about 1.8, and the corresponding BOAT3D run started at 2.3. Fridsma's 30 degree deadrise boat with its CG set back 74% porpoised at a Froude number of about 1.2 whereas the same BOAT3D run porpoised at about 1.8.

As shown in figures 12, 13, 14, and 20 above, BOAT3D simulated values have relatively good agreement with experimental trim on calm water for Froude numbers between 0.6 and 1.8 (typical speeds for utility and cruising planing boats), but there is a lack of agreement (Figure 20) with experiments of Katayama et al. at Froude numbers above 1.5.

## Comparison of Simulation Results With Data from Full-Sized Boat Sea Trials

Blount and Hankley (9) published a variety of data from various tests on various full-sized boats. Two of these were U. S. Navy patrol vessels which were tested in the open sea. The patrol boat denoted Craft I was an especially long and narrow boat and Craft II was more conventional. Craft I had double chines and Craft II conventional single chines.

Blount and Hankley provided photographs and cross-sectional shapes but did not state length or beam dimensions directly. It was stated that Craft I was tested in 4.6 foot significant wave height sea and Craft II in a 2.2 foot sea, and the ratios of the wave height to beam were given, so we could use these beam values. The authors mentioned the length-to-beam ratio for Craft I but only gave a range for Craft II, so we think that our length and beam are close to the correct values for Craft I, but the length for the shorter Craft II is only a rough estimate. Also, we estimated the displacement of each boat by adjusting the water lines in B3DBOAT so they resembled the photographs. We estimated typical radii of gyration and assumed a fully-developed (i.e. low steepness) random sea in each case.

The B3DBOAT drawings resulting from careful measurements of Blount and Hankley's drawings and the above-described assumptions are shown in Figure 21. BOAT3D simulation runs were made at various speeds and the results plotted superimposed with the full-sized boat experimental points from Blount & Hankley's figures 18 through 23.

For Craft I Figure 22 below shows the results for the mean of the 1/10th highest acceleration peaks at the CG, Figure 23 shows corresponding results at the bow, and Figure 24 shows the results for the mean of the 1/10th highest pitch amplitudes (relative to mean pitch). Figures 25 through 27 show corresponding results for Craft II.

In the comparison test published previously for BOAT3D V.4.0 and earlier, we did not perform analyses for Craft I, and we assumed a larger size for Craft II. For the current work we assumed the following:

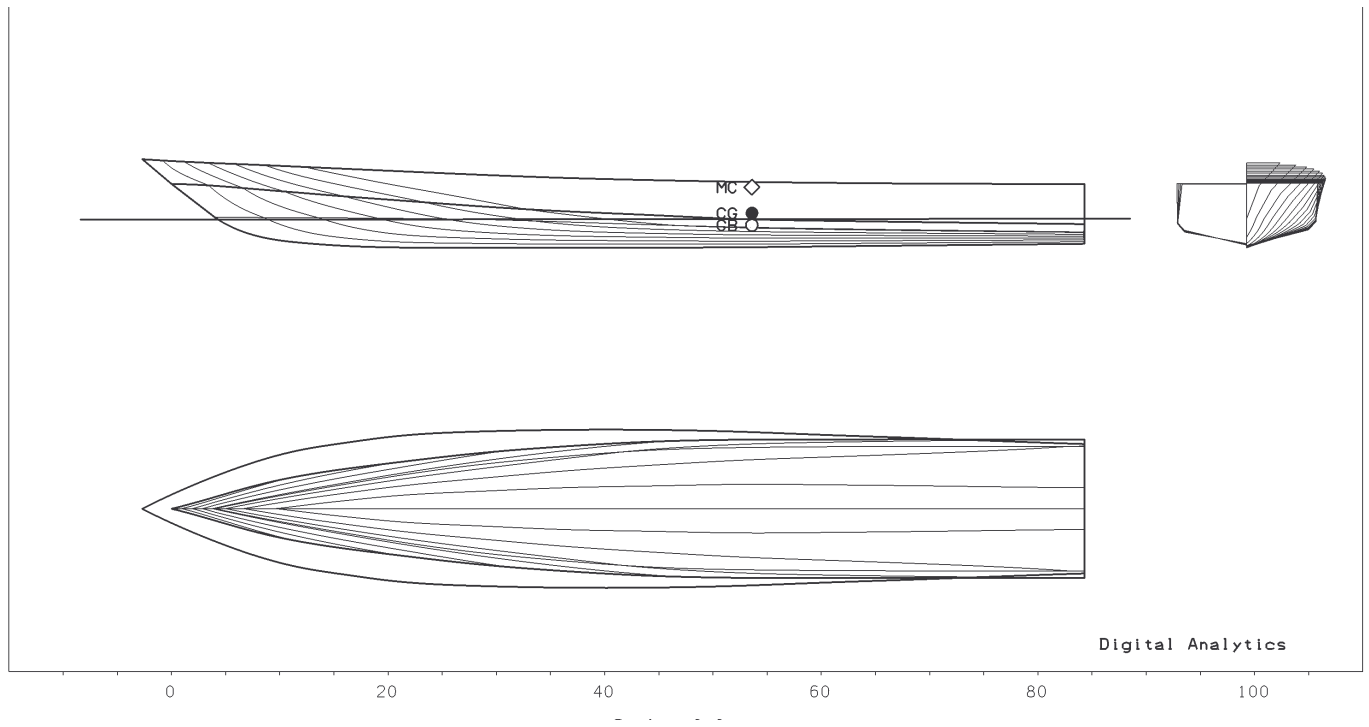
	Craft I	Craft II
LOA, ft	87	55
Max Beam, ft	14.5	14.6
Displacement, lb	77,000	40,000
Radius of Gyration, ft	24	15.8
Significant wave height	4.6	2.2
Wave steepness parameter	0.023	0.023

Only the Significant wave height appears explicitly in the original reference (9).

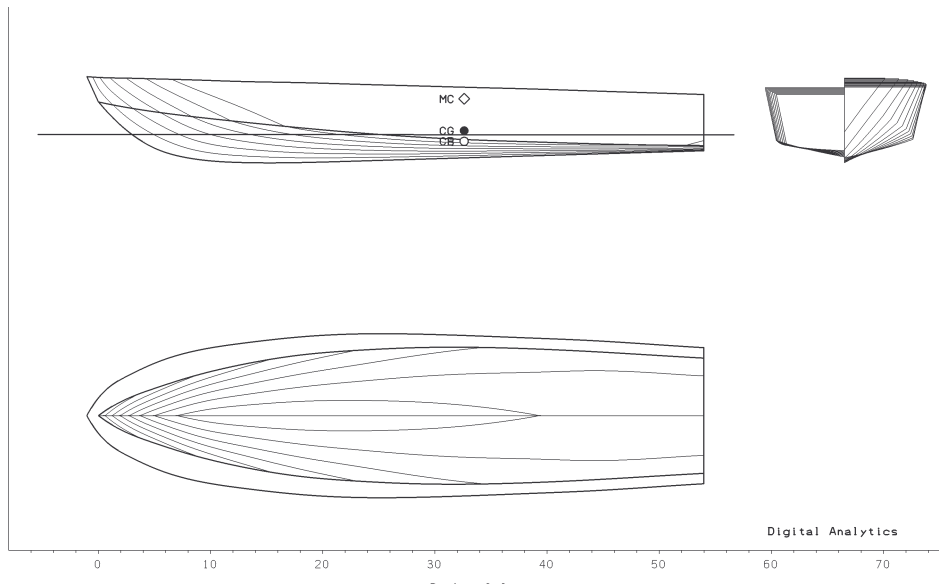
We used the default Random Sea generator as designed for BOAT3D by Peter Payne with the stated Significant wave height and Steepness parameters.

Figure 21 Lines Assumed for Patrol Boats Investigated by Blount and Hankley

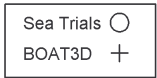
Craft I



Craft II



(Refer to the following to interpret figures 22 through 27)



Results for patrol boat "Craft I" with 4.6 ft significant wave height:

Figure 22 Vertical Acceleration at the CG

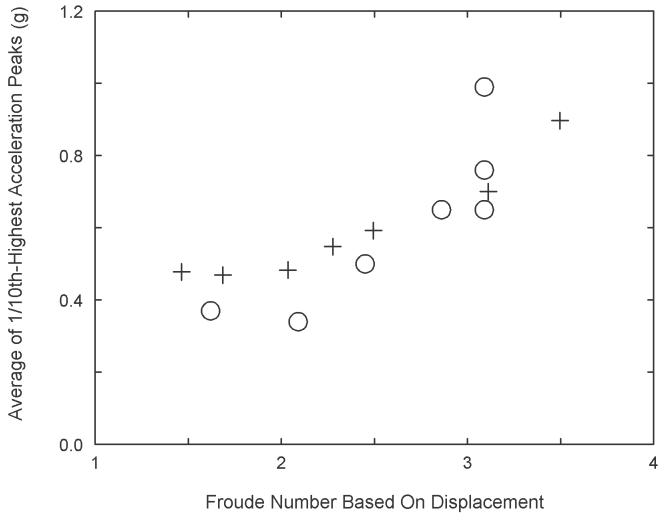


Figure 23 Vertical Acceleration at the Bow

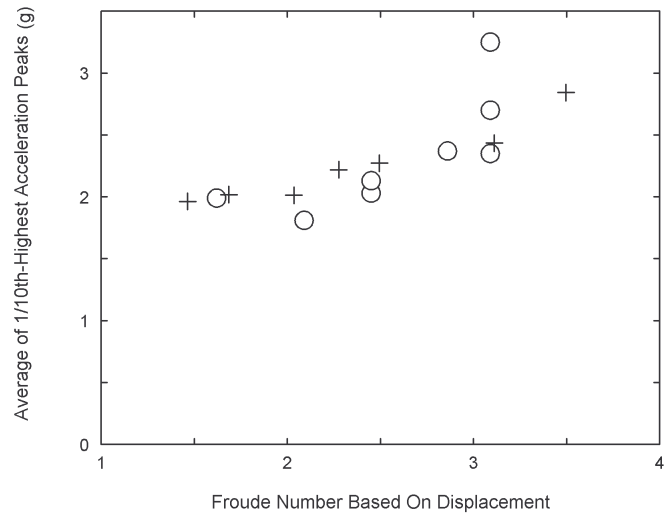
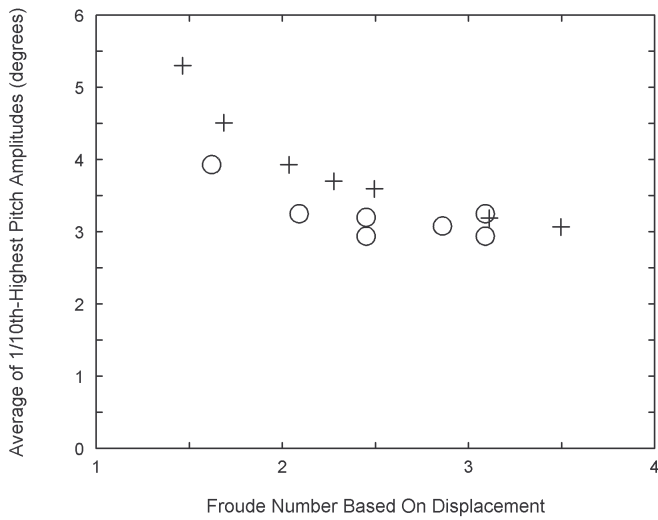


Figure 24 Pitch Amplitude



Results for patrol boat "Craft II" with 2.2 ft significant wave height: Note --

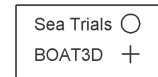


Figure 25 Vertical Acceleration at the CG

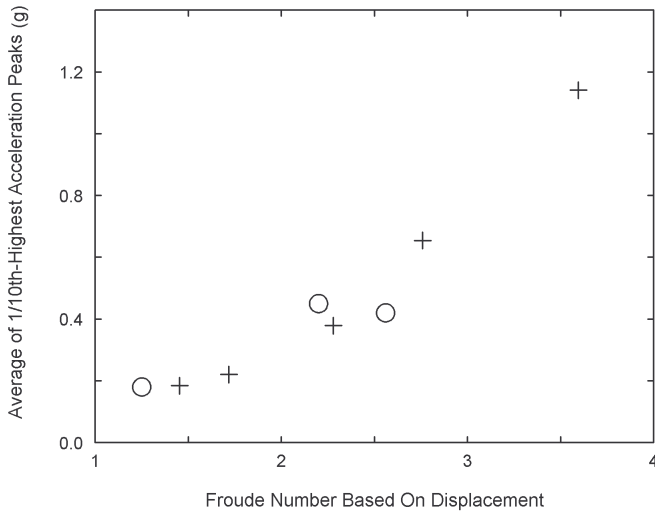


Figure 26 Vertical Acceleration at the Bow

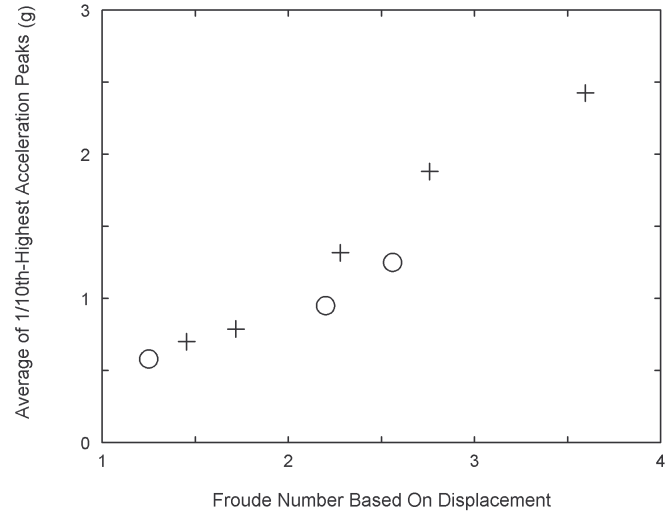
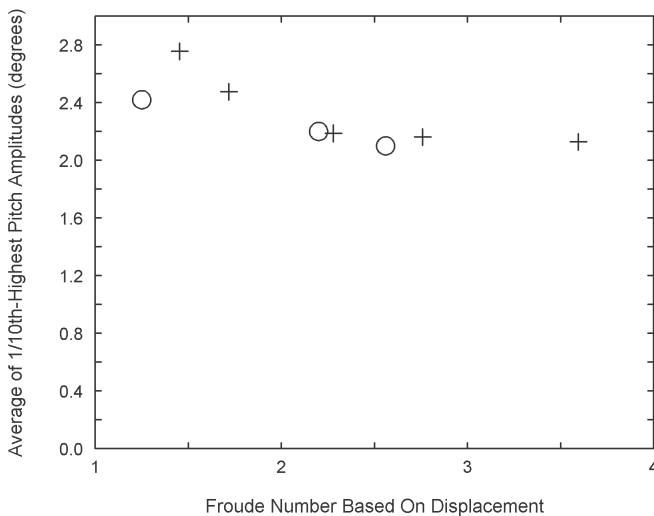


Figure 27 Pitch Amplitude



In contrast to the comparisons of BOAT3D results with uniform-wave tank tests, the patrol boat comparisons also test whether the 2-dimensional random wave generator in BOAT3D (developed by P. R. Payne with input from J. D. Pierson) can be employed successfully vis-a-vis real 3-dimensional sea waves, and this is supported by the results. It should be noted that both sea-trial and simulation values have an element of randomness in them from the randomness of the waves.

The Froude number used by Blount and Hankley is based on cube root of displaced volume, whereas the more-frequently-used measure of planing vs displacement is the Froude number containing water line length. The lowest length-based Froude number occurring for a BOAT3D run is 0.54 for Craft I and 0.60 for Craft II. As discussed above under the topic "Semi-Planing Froude Numbers", these runs qualify as "semi-planing" (or "semi-displacement"). As discussed, for the new version 4.2 we added "artificial low-speed damping" which Peter Payne designed into BOAT3D but which had been turned off before. This

was a necessary change to model behavior of Craft I, to reduce excessive pitching and slamming at low speeds and give reasonable acceleration values at low speeds. With this change less compromises are needed to get good agreement for Craft II than for previous BOAT3D versions. With the damping formulation now in BOAT3D, there is improvement in another low-speed case, C12 in the Fridsma comparisons above. Before introducing more damping the simulated trim amplitude was excessive.



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